Growth and the Cycle

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GROWTH AND THE CYCLE: CREATIVE DESTRUCTION VERSUS ENTRENCHMENT

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Abstract

Newly established firms often try to secure their market position by building up a base of loyal customers. While recessions may not destroy technological leadership, they may be harmful for such firm-customer relationships. Without such customer bases, these firms find themselves more vulnerable to attacks by competitors. We formulate this idea within an Aghion-Howitt-type model of creative destruction and discuss its implications for growth. In the context of this model, recessions might be good for growth since they weaken the incumbent firm's position, and thereby stimulate research by outside firms. The model allows for the extreme case where the leading firm can be so entrenched that growth ceases, unless a recession shakes up its customer base. We find a one-to-one relationship between the average growth rate and the cyclical variability, a U-shaped relationship between the average speed of building up good customer relationships and the average growth rate, and a positive relationship between the arrival rate of recessions and average growth. It is finally shown that an appropriate stochastic tax program can implement the social planner's solution. In some cases, general equilibrium effects may generate interesting results, conflicting with intuition from a partial equilibrium approach: we show that, in some cases, a social planner might want to subsidize research in order to discourage it.

1. Introduction

Technological breakthroughs are often not enough to strongly establish a firm in the market. It also needs further marginal improvements of the product according to customer needs, or to build up consumer recognition to secure its position. Building up such a position takes time. And while recessions may not destroy technological breakthroughs, they may be seen as disrupting such firm-customer relationships. Thus, firms who have

[1] Suggestions from Henri de Groot, Jos Jansen and participants of the ENTER Jamboree, January 1997, are greatfully acknowledged.

not yet built such relationships or whose relationships have gotten destroyed in a recession, find themselves more vulnerable to attacks by competitors than those that did. We formulate this idea within an Aghion-Howitt-type model of creative destruction (see Aghion and Howitt, 1992) and study its implications for growth. In particular, if the lead firm lacks an established position, the competitors' incentives for attacks are increased, leading to higher R&D efforts on their part in the hope of leapfrogging the leader. Thus, in this model, recessions are actually good for growth, since they encourage new creative destruction. Booms and established market leads, on the other hand, can in the extreme completely eliminate all desire for R&D, leading to complete entrenchment of the leader and to a stand-still in growth, until the next recession destroys the secure market lead.

This paper fits within the literature on creative destruction, initiated by Schumpeter (1942), and more recently by Segerstrom, Anant, and Dinopoulos (1990) and Aghion and Howitt (1992).[2] Economic growth is driven by the process of creative destruction, *i.e.* the introduction of new products by innovating firms and the replacement of incumbent market leaders. The contribution of this paper is that we distinguish fundamental innovations leading to creative destruction from marginal innovations that slow down the process of creative destruction. The marginal innovations in our model capture the build-up of a well-functioning firm-customer relationship. Strong market leaders with a loyal customer base can - at least partially - insulate themselves from the threat of leapfrogging by potential entrants. Secondly, along the lines of Caballero and Hammour (1994), we analyze the cleansing effect of recessions by assuming that established firm-customer relationships will be destroyed in a recession. In order to keep the analysis analytically tractable, we assume that marginal innovations and recessions are exogenous stochastic events. The intermediate firm cannot influence the probability of achieving a strong customer base. Likewise, recessions are interpreted as sudden disruptions of such loyal customer bases.

Related ideas have received some attention in the recent literature. Cheng and Dinopoulos (1993) construct a model of Schumpeterian growth driven by asymmetric technological opportunities in the form of high-cost high-quality breakthroughs and low-cost low-quality improvements. They assume that each product generation starts with a

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[2] An overview of Schumpeterian growth theory can be found in Dinoupolos (1996).

quality breakthrough, followed by a single improvement. The pattern of growth and fluctuations can then be described as a stationary market equilibrium in which R&D races alternate between a breakthrough and an improvement. Our approach is different in the sense that we allow incumbent firms to gain from experience: firms that are longer in the market are more likely to carry out marginal improvements, or to establish a loyal customer base. Stein (1997) develops a model of creative destruction in which firms compete on product quality and on distribution costs. A firm's innovation in product quality ultimately spills over to new firms, whereas distribution costs are taken to be firm-specific: incumbent firms have an advantage over their potential competitors when they can reduce distribution costs through loyalty of their customers. In line with our results, Stein finds that firm-specific learning-by-doing may discourage research activity and thereby reduce long-term economic growth. In contrast with Stein's analysis, we take account of the possibility that such firm-specific advantage may suddenly be disrupted by the event of a recession. Li (1996) analyzes an R&D-based growth model where the heterogeneous nature of technical progress is captured by distinguishing between fundamental and secondary innovations. Li underlines heterogeneity in the stock of knowledge, giving rise to the possibility of multiple equilibria. In our analysis we distinguish between fundamental knowledge which spills over to other firms, and firm-specific knowledge leading to a loyal customer base or to minor product improvements. Jovanovic and Rob (1990) explore the link between long-run productivity growth and the length and amplitude of business cycle fluctuations in the context of a model that formalizes the distinction between extensive and intensive search. Extensive search is directed at major breakthroughs, while intensive search aims to refine such fundamental breakthroughs. Caballero and Hammour (1996) analyze the timing, pace, and efficiency of job creation and destruction resulting from product and process innovation. While an efficient economy concentrates such job reallocation processes during recessions (because of the opportunity cost effect), incomplete contracting between labour and capital as well as transactional difficulties may decouple the synchronized pattern of creation and destruction, leading to technological "sclerosis". Economic efficiency can be restored through an appropriate mix of government policies. We also aim to design optimal tax policies that restore economic efficiency, but in our story such taxation scheme is shown to be state-contingent.

In the context of our model, recessions might be good for growth since they weaken the incumbent firm’s position, and thereby stimulate research by outside firms. The model allows for the extreme case, where the leading firm can be so entrenched that growth ceases unless a recession shakes up its customer base. We find a one-to-one relationship between the average growth rate and the cyclical variability, a U-shaped relationship between the average speed of building up good customer relationships and the average growth rate, and a positive relationship between the arrival rate of recessions and average growth. We do not view these claims as immediately testable empirical predictions of our model, however. Rather we like to think of our model as just analyzing one of many facets of economic fluctuations in isolation. For the same reason, we have abstained from attempting a serious calibration exercise. In our view, such a calibration exercise can only be done on the basis of a more complete, but thus also more complicated analysis of all the facets involved. We proceed along the following lines. Section 2 introduces the model, derives optimality conditions, and describes the equilibrium solution. Some interesting numerical examples are discussed in section 3 to describe some special features of the model. In particular, we study the possibility of entrenchment: strong market leaders can in the extreme completely eliminate incentives to carry out R&D by potential competitors, leading to complete entrenchment in the market and to a stand-still in economic growth. The model’s implications for growth and the business cycle are more elaborately discussed in section 4. Out-of-equilibrium dynamics will be considered in section 5. Since the equilibrium solution of the model is not an efficient solution, we will investigate the policy selected by a benevolent social planner in section 6. In section 7 it is shown that an appropriate stochastic tax program can implement the social planner’s solution. In some cases, general equilibrium effects may generate interesting results, conflicting with intuition from a partial equilibrium approach: we show that, in some cases, a social planner might want to subsidize research in order to discourage it. Briefly, the intuition is that the market leader has to pay taxes to finance these research subsidies. This may lower its value by a substantial amount, so that firms in the research sector expect substantially lower gains from innovative activity, and actually decide to undertake less research activity. If this sounds not yet convincing, we hope that it entices to read section 7 for a more extensive discussion and analysis. Finally, section 8 concludes.

2. The model

2.1 Environment

Consider an economy with three classes of tradeable objects: labour, a consumption good, and an intermediate good. Time is continuous. All markets are perfectly competitive, except for the intermediate goods market. The economy is populated with a continuum of infinitely-lived, representative agents. These agents choose contingency plans for lifetime consumption, evaluated at a constant rate of time preference $r>0$ and linear instantaneous utility. Thus, r is also the rate of interest.

The agents also supply labour. Labour supply is constant, inelastic, and normalized to unity. Two categories of labour are distinguished: unskilled labour, which can only be used in the production of the final good which is used for consumption, and skilled labour, which can be employed in research and in the intermediate sector.

Production takes place in two sectors: a competitive final goods sector and a monopolistic intermediate goods sector. Furthermore, there is a sector undertaking research. A firm in the competitive final goods sector hires unskilled labour m and purchases the amount x of the intermediate good to produce output y according to (leaving away the time subscript)

$$y = A_f F\left(\frac{x}{m}\right) m \tag{1}$$

where F is strictly increasing, strictly concave, and differentiable. The factor A_f is the current productivity of final goods production "embodied" in the intermediate good: more advanced intermediate goods allow final goods production firms to produce with higher total factor productivity. Normalizing the aggregate quantity of unskilled labour to unity, aggregate production is $y=A_f F(x)$.

The productivity A_f is thus intimately tied to the particular intermediate input x which is used, and which is sold by a monopoly. Fundamental innovations increase this productivity by a fixed factor γ. Therefore, the time profile of the productivity parameter is given by

$$A_f = A_0 \gamma^f \qquad \gamma > 1 \tag{2}$$

f=0,1,2,3,... denotes the fundamental innovation. A fundamental innovation brings about a new intermediate input allowing firms in the final goods sector to produce more efficiently. The new intermediate product renders existing ones obsolete. Thus, fundamental innovations replace the existing intermediate firm by a new monopoly in the now leading technology : economic growth is driven by creative destruction. It will be assumed that fundamental innovations occur randomly with Poisson arrival rate λn, where n is the flow of skilled labour used in research. The research sector itself is competitive, but a successful innovator can protect his fundamental innovation by a patent which he can use to monopolize the intermediate sector. According to eq. 2, the knowledge incorporated in a new intermediate input ultimately spills over to new firms: innovators stand on the shoulders of giants.

The production function of the intermediate good x is linear,

$$x = BL \tag{3}$$

L is the flow of skilled labour used in the intermediate sector. To capture the idea that an intermediate firm needs further marginal improvements of the product according to customer needs, or to build up consumer recognition to secure its position, we introduce the parameter B. For simplicity, we assume that B can only take two values: $B \in \{\delta, 1\}$. If $B=\delta>1$, the intermediate firm is a strong market leader, but if B=1 the monopolist is a weak market leader.

According to eq. 3, the strength of the monopolist is reflected in parameter B. It should however be noted that there is no formal difference between technical improvements in the final goods sector or in the intermediate goods sector. Total factor productivity of the final goods sector is determined by the current technology of the intermediate monopolist. Thus, the establishment of strong market leadership by the intermediate monopolist also translates into increased productivity in final goods production.

Our assumption of a "one intermediate good economy" is an extreme case. The opposite extreme would be an economy with many monopolists in different industries. Fluctuations between weak and strong market leadership at the level of the intermediate firms will be washed out via the law of large numbers. However, our assumption of one intermediate monopolist would still be valid when labour movements between sectors are "slow".

Newly established firms can secure their market position by building up a base of loyal customers. Experience from being in the market can turn a weak market leader into a strong one. We specify this learning-by-doing as an exogenous stochastic Markov process, where μ is the Poisson arrival rate for a weak monopolist to become strong. To put it differently, we simply assume that older firms are more likely to have a loyal customer base than young ones (*ceteris paribus*).

While recessions may not destroy technological breakthroughs, they may be seen as disrupting such firm-customer relationships. The event of a recession will consequently turn a strong monopolist into a weak one. Specifically, let us assume that the Poisson arrival rate of a recession is given by ν.

It is probably more realistic to assume that the intermediate firm can at least partly influence the probability of becoming a strong market leader, *e.g.* via active marketing campaigns or through additional investments in the product or its distribution channels. However, the establishment of a loyal customer base is also likely to be affected by some randomness. Firms can try to promote their products, but the success or failure of such conduct is also determined by unpredictable or unexpected factors. In order to keep the analysis analytically tractable, we take an extreme position and assume that marginal innovations are purely exogenous stochastic events. The intermediate firm cannot influence the probability of becoming strong. Likewise, recessions may relate to a broad range of events, including negative productivity changes, disturbances on the demand side, or difficulties with regard to market interactions between relevant parties. Our interpretation of recessions as sudden disruptions of such loyal customer bases is thus one out of different possibilities.

To rule out the possibility of strategic behaviour, we henceforth assume that $\gamma>\delta$: the size of drastic innovations in the productivity of final output is larger than the size of marginal innovations in the intermediate goods sector from learning-by-doing.

Consider a firm which has made the f-th innovation. During its lifetime, the intermediate firm can find itself in two different states. In the first state, the incumbent firm is a weak market leader. In the second state, the incumbent firm is strong. After some random time span, the incumbent monopolist will be superseded by a new intermediate firm through the event of the f+1-th fundamental innovation.

The various transitions across states initiated by fundamental and marginal innovations can be tabulated as follows. Denoting the state of the f-th intermediate firm by i and the state of the new firm by j, we have the following transition structure during a small time interval dt:

f		$f+1$	transition probability
$i=1$	$\rightarrow$	$j=1$	$\lambda n_f^{(1)} dt$
$i=2$	$\rightarrow$	$j=1$	$\lambda n_f^{(2)} dt$

Implicitly, we have assumed the labour input into research to depend only on the index f of the innovation, *i.e.* to be constant in the time interval during which the f-th but not the f+1-th innovation has been undertaken. In the further analysis we will see that this is justifiable in equilibrium.

Equivalently, denoting the state of the intermediate firm at t ($t+dt$) by i (j), we have the following transition structure in case of incremental leaps:

t		$t+dt$	transition probability
$i=1$	$\rightarrow$	$j=2$	μdt
$i=2$	$\rightarrow$	$j=1$	νdt

An equilibrium are lists of firm values ($V_f^{(i)}$), research labour ($n_f^{(i)}$), intermediate goods production ($x_f^{(i)}$), intermediate goods labour ($L_f^{(i)}$), wages for skilled labour ($w_f^{(i)}$), and profits ($\pi_f^{(i)}$) for f=0,1,2,... and i=1,2 so that at each level f and each state i,

(i) the current intermediate goods monopolist maximizes instantaneous profits, given wages $w_f^{(i)}$,

$$\pi_f^{(i)} = \max_{\{x_f^{(i)}, L_f^{(i)} \leq N\}} A_f F'(x_f^{(i)}) x_f^{(i)} - L_f^{(i)} w_f^{(i)}$$
$$\text{s.t. } x_f^{(i)} = B^{(i)} L_f^{(i)}$$

Note that we have substituted in the demand function for the intermediate good sector, resulting from the final goods production sector.

(ii) The firm value is given by

$$V_f^{(i)} = \sum_{j=1}^{2} \int_{t=0}^{\infty} e^{-rt} \pi_f^{(j)} P_f^{(i)}(t,j) dt,$$

where $P_f^{(i)}(t,j)$ is the probability that the current intermediate good monopolist is still the market leader t time units from now, and is in state j then.

(*iii*) Given the wage $w_f^{(i)}$, the competitive R&D firms maximize the instantaneous profits from R&D, calculated as the instantaneous value of a successful innovation times its instantaneous probability, minus the instantaneous wage costs,

$$\max_{\{n_f^{(i)} \geq 0\}} V_{f+1}^{(1)} \lambda n_f^{(i)} - w_f^{(i)} n_f^{(i)}$$

(*iv*) The market for skilled labour clears,

$$N = L_f^{(i)} + n_f^{(i)}$$

where N denotes the mass of skilled individuals.

2.2 The maximization problems

Having finished the description of the economy, we now turn to the optimality conditions. At each instant in time, the monopolist can be in two different states, as described above. Therefore, two Bellman equations need to be constructed. For instance, when the intermediate firm is currently in the weak state, it makes the instantaneous profit, $\pi_f^{(1)}$. The probability of still being a monopolist after a small time interval dt has elapsed is equal to $1-\lambda n_f^{(1)}dt$. Within this interval, the (unconditional) probability of a marginal innovation is μdt. By the event of a marginal innovation the monopolist switches to the second state, and the firm's value is given by $V_f^{(2)}$. With probability $1-\mu dt-\lambda n_f^{(1)}dt$ the firm does not make the transition to state 2 during the time interval but is still the market leader, so that its value is still given by $V_f^{(1)}$. Proceeding along these lines, the Bellman equations can be written in the form (details can be found in the Technical Appendix)

$$\begin{bmatrix} r+\lambda n_f^{(1)}+\mu & -\mu \\ -\nu & r+\lambda n_f^{(2)}+\nu \end{bmatrix} \begin{bmatrix} V_f^{(1)} \\ V_f^{(2)} \end{bmatrix} = \begin{bmatrix} \pi_f^{(1)} \\ \pi_f^{(2)} \end{bmatrix} \tag{4}$$

Or, abbreviated,

$$XV = \pi \tag{5}$$

X is the 2×2 matrix from eq. 4, V=[$V^{(1)}$ $V^{(2)}$]', and π=[$\pi^{(1)}$ $\pi^{(2)}$]'.

Consider, next, the research sector. A potential entrant successfully doing research will start in state 1. The instantaneous expected gain for the f-th innovator when the current market leader is in state i is thus equal to $V_{f+1}^{(1)}\lambda n_f^{(i)}dte^{-rdt}$. The instantaneous cost of doing research is $w_f^{(i)}n_f^{(i)}dt$, where $w_f^{(i)}$ denotes the wage of skilled labour. An optimizing R&D firm chooses $n_f^{(i)}$ so as to equalize both terms, taking V and w as given. It follows that

$$w_f^{(i)} \geq V_{f+1}^{(1)}\lambda, \quad n_f^{(i)} \geq 0 \tag{6}$$

with at least one equality.

Firms in the final goods sector choose $x_f^{(i)}$ to maximize profits $A_f F(x_f^{(i)})$-$p_f^{(i)}x_f^{(i)}$, taking the relative price of the intermediate good $p_f^{(i)}$ as given. The first order condition for firms in the final goods sector is thus given by

$$A_f F^{\prime}(x_f^{(i)}) = p_f^{(i)} \tag{7}$$

Consequently, the intermediate firm chooses $x_f^{(i)}$ to maximize [$A_f F'(x_f^{(i)})$-$w_f^{(i)}/B^{(i)}$]$x_f^{(i)}$. The optimality condition is given by

$$\omega_f^{(i)} = B^{(i)}\{F^{\prime\prime}(x_f^{(i)})x_f^{(i)} + F^{\prime}(x_f^{(i)})\} \tag{8}$$

where $\omega_f^{(i)}=w_f^{(i)}/A_f$ is the productivity-adjusted wage.

2.3 Stationary equilibrium

In a stationary equilibrium, variables do not depend on the state f. Unless otherwise indicated, we concentrate in the sequel on interior equilibria, while the situation where no research is undertaken by outside firms and the incumbent monopolist is entrenched and completely insulated from creative destruction will be discussed as a special case in section 3 (Example 4). At each instant in time, the economy only needs to decide upon the allocation of skilled labour between manufacturing and research.

To determine $\omega^{(i)}$, we define $\tilde{V}^{(i)}=V_f^{(i)}/A_f$ and make use of the following Proposition:

Proposition 1:
In stationary interior equilibrium it must hold that $\omega^{(1)}=\omega^{(2)}=\omega$.

Proof:
From eq. 6 and the transition structure for fundamental innovations it follows that $w_f^{(i)}=w_f$, or $\omega^{(i)}=\omega$. □

Notice that this proposition only holds for interior equilibria. The proposition says that the productivity-adjusted wage of skilled labour is constant across both states. That is, skilled workers do not benefit from marginal innovations within the intermediate firm.

Using a Cobb-Douglas production function, $F(x)=x^{\alpha}$, we can readily express the solution in terms of ω:

$$x^{(i)} = \left[\frac{\alpha^2 \boldsymbol{B}^{(i)}}{\omega^{(i)}}\right]^{\frac{1}{1-\alpha}}; \quad \tilde{\pi}^{(i)} = \frac{1-\alpha}{\alpha}\frac{\omega^{(i)}x^{(i)}}{\boldsymbol{B}^{(i)}}; \quad \tilde{p}^{(i)} = \frac{\omega^{(i)}}{\alpha \boldsymbol{B}^{(i)}}$$

where $\tilde{\pi}^{(i)}=\pi_f^{(i)}/A_f$ and $\tilde{p}^{(i)}=p_f^{(i)}/A_f$. For the Cobb-Douglas case, we obtain the stationary equilibria of the model from the following Proposition:

Proposition 2:
There are in general two solutions for ω, given by

$$\bar{\omega}_{1,2} = \left(\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}\right)^{1-\alpha}$$

where a, b, and c are stated in the Technical Appendix.

A proof is given in the Technical Appendix.

3. Results

Using the Cobb-Douglas specification, we turn to some numerical examples to illustrate the stationary equilibrium (or equilibria). As a baseline, we more or less arbitrarily pick the following values: r=0.1; A_0=$B^{(1)}$=N=1; $B^{(2)}$=1.2; α=0.5; γ=1.4; δ=1.2.

Example 1: Aghion-Howitt

We first discuss the model's equilibrium solution in the absence of learning-by-doing and recessions. By setting μ and ν equal to zero, we effectively are back in the Aghion-Howitt world. λ is set at 0.15. Although the fundamental quadratic from Proposition 2 delivers two equilibrium values for ω, only the "positive" root is economically meaningful (more precisely, only the "positive" root gives a non-negative research intensity). In this example, 31% of the skilled labour force is engaged in research activity.

Example 2: Learning-by-Doing

Next we allow intermediate firms to strengthen their market position by building up a base of loyal customers. We pick μ=0.5 (leaving other parameters equal). That is, we allow for the possibility of marginal innovations and assume that marginal leaps are more likely to take place than fundamental breakthroughs. We find $n^{(1)}$=0.40 and $n^{(2)}$=0.28. Since $n^{(1)}>n^{(2)}$ and n is positively related to the arrival rate of fundamental innovations, we can refer to state 1 (no marginal innovation) as the "high growth equilibrium" and state 2 (with marginal innovation) as the "low growth equilibrium". To put it differently, the creation of a loyal customer base by the intermediate firm discourages research activity by potential entrants, and thereby tends to lower economic growth. What is at work here, is a substitution effect: an intermediate firm making the transition to strong market leadership will set a lower price for its product, so that the final goods sector increases the demand for these intermediate inputs. The increased marginal product of workers in producing intermediate goods is not enough to meet the higher demand, so that more skilled workers need to be allocated to the manufacturing sector. Labour market clearing is established through reliefing skilled workers from the research sector, since the expected pay-off from R&D activity falls.

Example 3: Learning-by-Doing and Recessions

In the third example, we consider the possibility that strong firm-customer relationships are destroyed in a recession. The flow probability of recessions, ν, is set at 0.2. With these parameter values, we have assumed that agents expect recessions to take place less often than marginal innovations, but more frequent than fundamental innovations. In equilibrium we have $n^{(1)}$=0.38 and $n^{(2)}$=0.26. Although the introduction of recessions tends to reduce research intensity in both states of the economy, one cannot immediately assess the overall effect on economic growth. To see this, realize that in the presence of recessions the economy will spend more time in the high growth equilibrium. We will discuss the relationship between growth and recessions in greater depth in section 4 below.

Example 4: Entrenchment

We finally turn to the possibility of entrenchment: strong market leaders might completely eliminate all desire for R&D, leading to complete insulation of the incumbent monopolist from the process of creative destruction and to a stand-still in growth. Such a scenario will emerge from our model when (for instance) fundamental innovations occur less frequent. For λ=0.08 (and μ=0.5, ν=0.2) we have the knife-edge case and find $n^{(1)}$=0.17 and $n^{(2)}$=0. Marginal innovations lead to a stop of research activity and economic growth. Only the extricating event of a recession can bring the economy out of such a "no-growth trap" back to a process where firms try to leapfrog each other.

4. Economic growth and the cycle

We will now discuss the model's implications for economic growth and the cycle. In order to calculate the average growth rate in the economy, we need to find the expected fraction of time that the economy spends in each state. From a society's perspective, we have the following transition scheme between both states during a small time interval *dt*:

t		$t+dt$	transition probability
i=1	$\rightarrow$	j=1	$1-\mu dt$
i=1	$\rightarrow$	j=2	μdt
i=2	$\rightarrow$	j=1	$(\nu+\lambda n^{(2)})dt$
i=2	$\rightarrow$	j=2	$(1-(\nu+\lambda n^{(2)}))dt$

To see this, consider first an intermediate monopolist which is currently in state 1. With probability $\lambda n^{(1)}dt$ this monopolist will be superseded by a new intermediate firm through the event of a fundamental innovation during that time interval. This new intermediate firm will start operation in state 1. A monopolist currently in state 1 will make the transition to state 2 during dt with probability μdt. Thus, an incumbent firm which is in state 1 will maintain its current position during a small time span dt with probability $1-\lambda n^{(1)}dt-\mu dt$. From a society's perspective, we thus find that the probability of the economy still being in the first state after dt has elapsed is equal to $1-\mu dt$ whereas with probability μdt the economy has moved to the second state. Likewise, consider an intermediate monopolist which is currently in state 2. With probability $\lambda n^{(2)}dt$ this monopolist will be replaced by a new intermediate firm through the event of a fundamental innovation during that time span. Again, this new intermediate firm will start operation in state 1. A monopolist currently in state 2 will face a recession and fall back to state 1 during dt with probability νdt. Consequently, an incumbent firm which is in state 2 will maintain its current position during a small time span dt with probability $1-\lambda n^{(2)}dt-\nu dt$. From a society's perspective, we thus find that the probability of the economy still being in the second state after dt has elapsed is equal to $1-\lambda n^{(2)}dt-\nu dt$ whereas with probability $\nu dt+\lambda n^{(2)}dt$ the economy has moved back to the first state.

Denoting the stationary probability that the firm is in state i by $q^{(i)}$ and using that $q^{(2)}=1-q^{(1)}$, we have in stationary flow $q^{(1)}=q^{(1)}(1-\mu dt)+(1-q^{(1)})(\nu+\lambda n^{(2)})dt$, or

$$q^{(1)} = \frac{\nu+\lambda n^{(2)}}{\mu+\nu+\lambda n^{(2)}}; \quad q^{(2)} = \frac{\mu}{\mu+\nu+\lambda n^{(2)}} \tag{9}$$

Naturally, the firm will never become a strong market leader when $\mu=0$. In Aghion and Howitt (1992) the average growth rate in the economy equals $\lambda.n.\ln(\gamma)$, where $\lambda.n$ is the arrival rate of fundamental innovations. A similar expression can easily be derived by weighting the research intensity in each state, $n^{(1)}$ and $n^{(2)}$, by the expected fraction of time that the economy spends in each state, determined by eq. 9. The average growth rate

(AGR) is thus found to be given by

$$AGR = \frac{\nu n^{(1)} + \lambda n^{(1)} n^{(2)} + \mu n^{(2)}}{\mu + \nu + \lambda n^{(2)}} \lambda \ln\gamma \tag{10}$$

Following a similar methodology, the variance of the rate of economic growth (VGR) can be expressed as

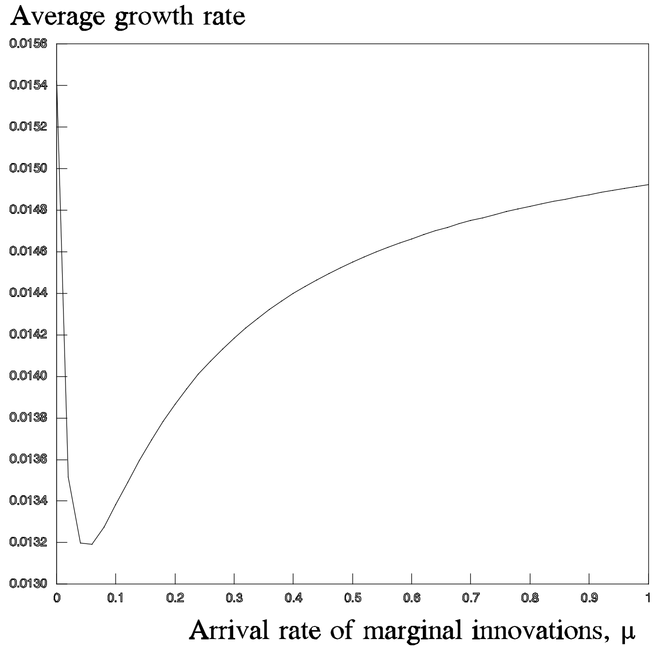
$$VGR = \frac{\nu n^{(1)} + \lambda n^{(1)} n^{(2)} + \mu n^{(2)}}{\mu + \nu + \lambda n^{(2)}} \lambda (\ln\gamma)^2 \tag{11}$$

The ratio of AGR over VGR is constant and equal to $1/\ln\gamma$: the average growth rate and the variance of the economy's growth rate are thus related in a linear fashion. Empirical evidence of such positive interaction between economic growth and the cycle is documented by Kormendi and Meguire (1985), and Grier and Tullock (1989).

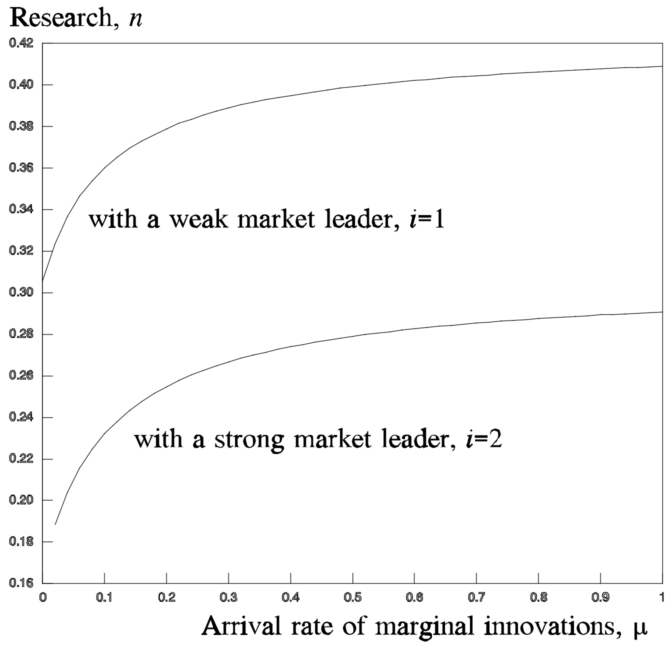
Having determined the economy's average growth rate and cyclical variability, we next turn to an evaluation of the effect of learning-by-doing and recessions on this growth rate. First we vary the speed μ of a marginal innovation, *i.e.* the effect of learning-by-doing, within the closed unit interval and study its implications for growth and research in Figure 1. Panel (a) shows a kind of U-shaped relation between the economy's average growth rate and the flow probability of marginal innovations: an increase in μ will tend to lower economic growth when firms need a relatively long time to learn about their customers' needs, whereas an opposite relation is found when firms learn fast. Panel (b) and (c) of Figure 1 explain the intuition behind this result. A strong market leader discourages R&D activity by potential entrants by increasing its expected lifetime. As μ is increased, firms tend to spend more time in the strong state, as Panel (c) shows. This discourages R&D. Call this the "discouragement-effect". On the other hand, research intensity $n^{(i)}$ is a positive function of the flow probability μ of marginal innovations: the prospect of being a strong market leader during a larger fraction of its lifetime increases the expected gains from fundamental innovations, and thereby stimulates research activity. Let us refer to this as the "reward-effect". Overall, the discouragement-effect dominates the reward-effect when firms need a long learning period, whereas the opposite holds when firms learn fast, leading to the observed U-shaped relation between AGR and μ.

We secondly vary the arrival rate of recessions, ν, within the closed unit interval (setting μ=0.5) and study its implications for growth and research in Figure 2. Panel (a) shows a positive relation between the economy's average growth rate and this arrival rate: an increase in ν will stimulate economic growth. Panel (b) and (c) again show that a strong market leader discourages R&D activity by potential entrants; $n^{(2)}$ is smaller than $n^{(1)}$ over the whole relevant domain. An increase in the arrival rate of recessions makes it more likely that the market leader is weak, encouraging R&D: this is the discouragement-effect "in reverse". This should increase growth. On the other hand, research activity is a negative function of the arrival rate of recessions; the intuition being that the prospect of losing strong market leadership earlier decreases the expected gains from fundamental innovations. This reward-effect "in reverse" decreases growth. Overall, the discouragement effect "in reverse" dominates.

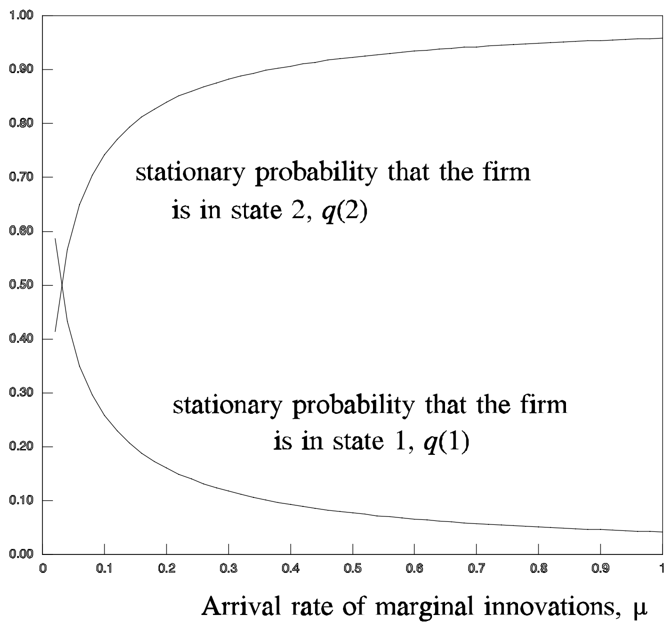
Panel (a)



Panel (b)



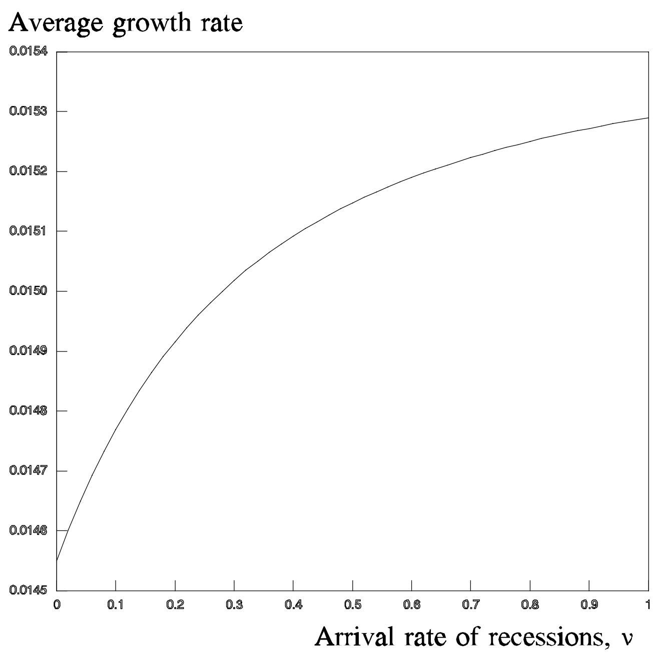
Panel (c)



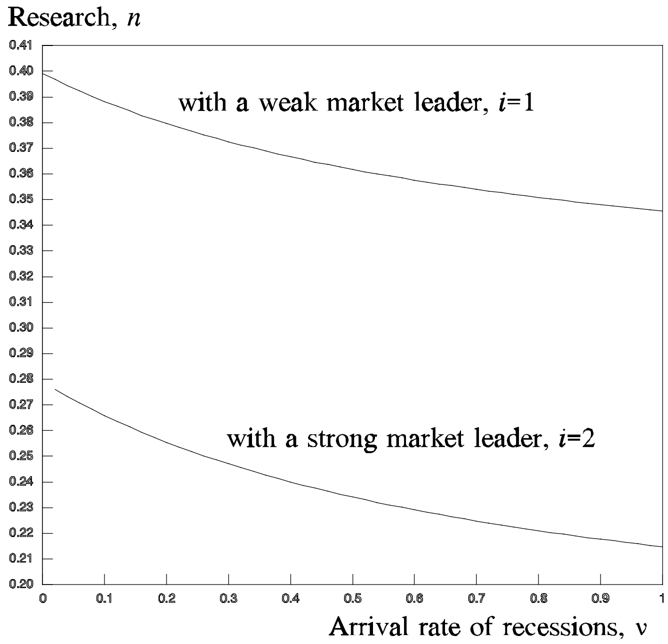
Note: i=1 denotes weak marketleadership; i=2 denotes strong marketleadership.

Figure 1: Effect of learning-by-doing on economic growth and research.

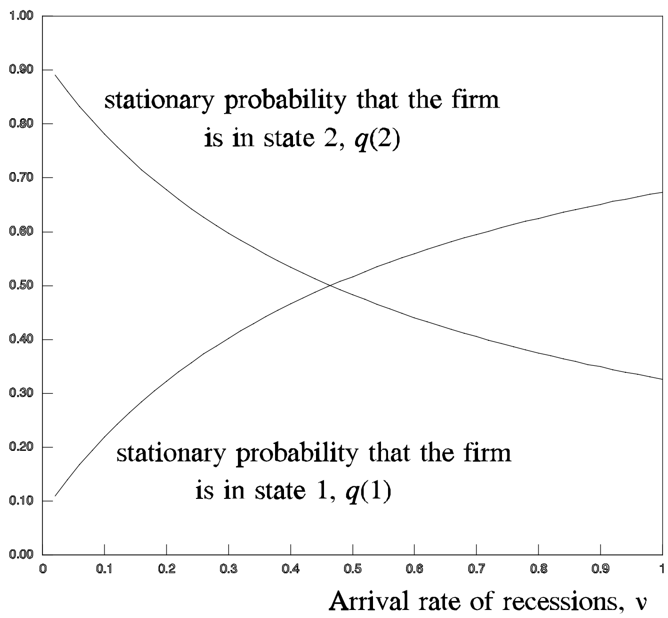
Panel (a)



Panel (b)



Panel (c)



Note: i=1 denotes weak marketleadership; i=2 denotes strong marketleadership.

Figure 2: Effect of recessions on economic growth and research.

5. Dynamics

The next step in our analysis is an investigation of the dynamics of our economy. To that end, we follow Aghion and Howitt (1992) by calculating the marginal cost and marginal benefit of research activity. For the two equilibria in our economy it should hold that

$$\begin{bmatrix} \tilde{\omega}(.)/\lambda \\ \tilde{\omega}(.)/\lambda \end{bmatrix} = X^{-1}\gamma \begin{bmatrix} \tilde{\pi}(.) \\ \tilde{\pi}(.) \end{bmatrix} \tag{12}$$

The marginal cost of doing research on the LHS of this expression follows from eq. 6, and the definition of ω. The marginal benefit of doing research on the RHS is determined from eq. 4 (dated at f+1), and the definition of $\tilde{\pi}$.[3]

Both equilibria are graphically illustrated in Figure 3. The marginal cost curves are denoted by *MC* and are upward sloping. The downward sloping curves *MB* represent marginal benefits from research effort. Point *H*(igh) [*L*(ow)] denotes the economy's equilibrium position when the intermediate firm is in state 1 [2]. Since $n^{(1)}>n^{(2)}$, the probability of fundamental innovations, and thereby the rate of creative destruction, will be higher in point *H*.

Since both recessions and fundamental innovations bring the economy back in state 1, we can consider point *H* as the attractor of the system. To evaluate the stability of the attractor we proceed along the following lines. It follows from eq. 5 that

$$\tilde{V} = X^{-1}\tilde{\pi} \tag{13}$$

Substituting the stationary equilibrium expressions for $n^{(i)}$ and $\tilde{\pi}^{(i)}$ in terms of $\tilde{V}^{(i)}$ into the latter expression gives

---

[3] Notice that when μ=ν=0 equation 12 simplifies to

$$\frac{\tilde{\omega}(.)}{\lambda} = \frac{\gamma\tilde{\pi}(.)}{r+\lambda n_{f+1}}$$

which is identical to eq. 3.1 in Aghion and Howitt (1992) with a linear research technology.

$$\begin{bmatrix} \tilde{V}_f^{(1)} \\ \tilde{V}_f^{(2)} \end{bmatrix} = \Phi(\tilde{V}_{f+1}^{(1)}) \tag{14}$$

where Φ is a nonlinear function, stated precisely in the Technical Appendix. The steady state is locally stable, if $D\Phi_1>1$.

In the Technical Appendix we calculate that derivative, but it is hard to analyze analytically. Using parameter values from Example 3, we find $D\Phi_1=5.97$. Thus, for these parameters, the economic system follows a stable gradual adjustment trajectory, before settling down in equilibrium.

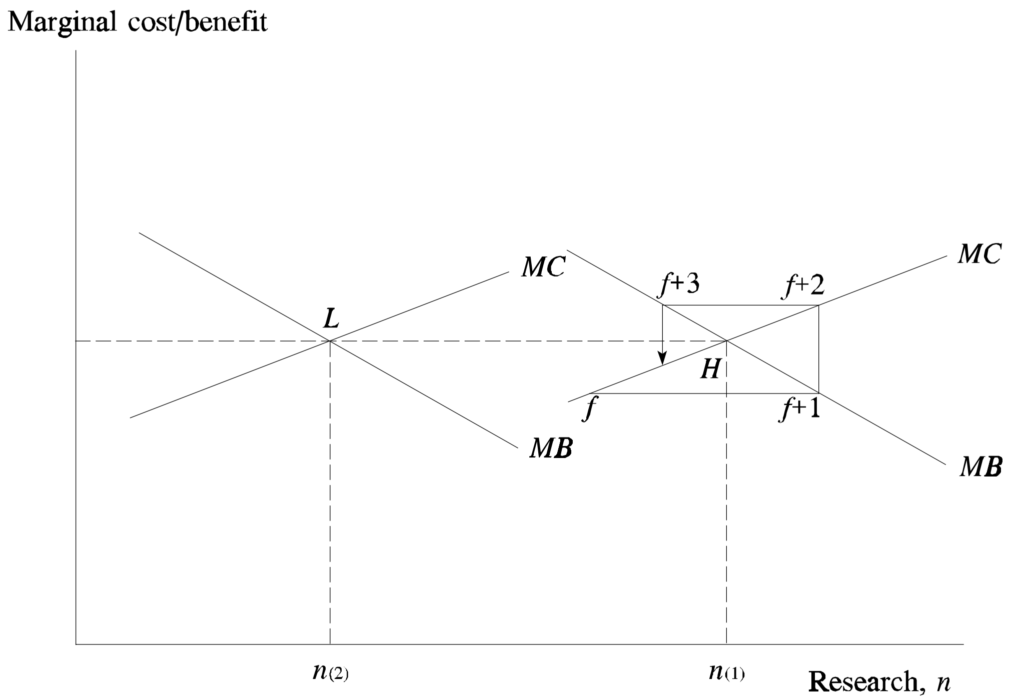


Figure 3: Dynamics.

6. Social planner

The balanced growth rate in a market economy may not be optimal from a society's point of view because of two external effects and an additional distortion. Firstly, intermediate firms cannot fully appropriate the rents generated by their fundamental innovations: the new technology ultimately spills over to other firms. Because of this intertemporal spillover effect, research activity and economic growth tend to be too low under laissez faire. On the other hand, innovating firms do not internalize the destruction of rents from leapfrogging the incumbent monopolist. This business-stealing effect will tend to overemphasize the benefits from research. Finally, the intermediate goods producer chooses its quantity monopolistically, possibly distorting the first best solution. In addition to these market imperfections, we want to raise the question whether strong firm-customer relationships are socially desirable or not. As we have seen, the discouragement effect that strong market leaders exert on potential entrants will lead to less R&D activity in the economy. However, the fact that strong market leaders can appropriate a larger share of the social value of their innovation since they can partly shelter from the threat of being leapfrogged by a new entrant, will encourage research activity.

The objective of a social planner is to choose R&D labour and thus quantities of the intermediate good in order to maximize the expected present value of consumption, subject to the constraints of feasibility. The social planners problem can be written as a dynamic program, which can be rewritten as (*cf.* the Technical Appendix)

$$\begin{bmatrix} r+\lambda n^{(1)}(1-\gamma)+\mu & -\mu \\ -\nu-\lambda n^{(2)}\gamma & r+\lambda n^{(2)}+\nu \end{bmatrix} \begin{bmatrix} U^{(1)} \\ U^{(2)} \end{bmatrix} = \begin{bmatrix} F(\boldsymbol{B}^{(1)}(N-n^{(1)})) \\ F(\boldsymbol{B}^{(2)}(N-n^{(2)})) \end{bmatrix} \tag{15}$$

where $U^{(i)}$ is the utility level when the economy is in state i. Compare eq. 15 to eq. 4 for the firm values in equilibrium. The upper left hand element of the matrix in eq. 15 contains the additional term $-\lambda n^{(1)}\gamma$ compared to eq. 4, reflecting the fact that R&D is valuable to the social planner, but not to the existing firm. The same holds true for the term $-\lambda n^{(2)}\gamma$ in the lower left hand element of that matrix.

Inverting the 2×2 matrix in eq. 15, and weighting lifetime utility in each state by the average fraction of time that the economy will spend in each state (*cf.* eq. 9), we finally express lifetime utility as

$$U = \sum_{i=1}^{2} \boldsymbol{q}^{(i)} U^{(i)} = \Xi(n^{(1)}, n^{(2)}) \tag{16}$$

where Ξ is a function, stated precisely in the Technical Appendix.

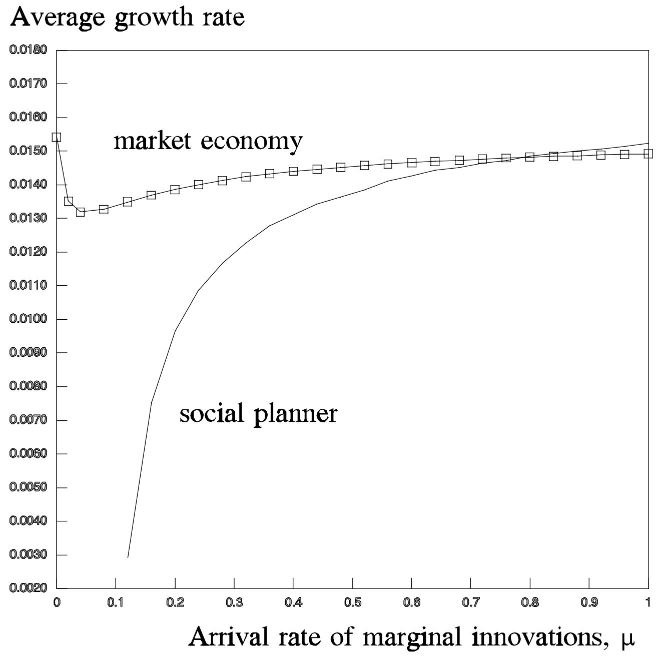
An optimizing social planner would select $n^{(i)}$ such that

$$\frac{\partial \Xi}{\partial n^{(1)}} = \frac{\partial \Xi}{\partial n^{(2)}} = 0 \tag{17}$$

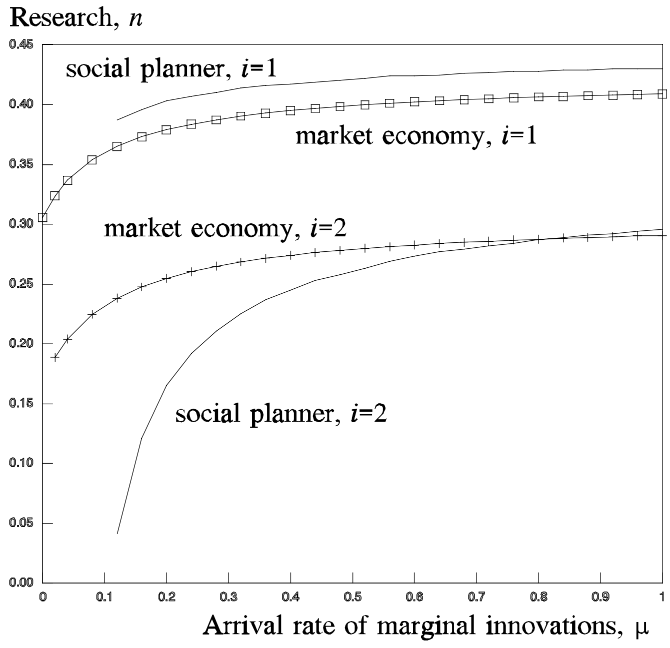
Since we were not able to obtain an equilibrium solution in closed-form, we resort to numerical simulations to discuss the effect of learning-by-doing and recessions on the economy's growth rate. The implications for growth and research from variation in μ within the closed unit interval are illustrated in Figure 4. Panel (a) shows a positive relation between the economy's average growth rate and the arrival rate μ of marginal innovations for values of μ exceeding (say) 0.1. The intuition behind this result and why and how it differs from Figure 1 can most easily be developed with the help of Panel (b) and Panel (c) of Figure 4. In the second Panel we show the optimal research program that a social planner would implement. It shows that the social planner allocates more workers to research if the market leader is strong. Intertemporal reallocations of skilled labour between production and research activities are intensified compared to the decentralized equilibrium situation. What is at work here, is that the gains related to a particular state of the economy are optimally used. An economy can better reallocate skilled workers from production towards research activity when the incumbent monopolist is weak, in order to fully exploit the temporary lower opportunity costs in terms of production forgone. Likewise, during a boom when the market leader is particularly good at producing intermediate inputs, one can better concentrate efforts in this direction, by relieving employees from research activity and allocating these workers to the monopolistic firm. Panel (b) also shows that research activity is intensified when μ is increased: the prospect of being a strong market leader during a larger fraction of its lifetime increases the expected gains from fundamental innovations, so that it is optimal to allocate more labour to research activity. Two additional comments are in order. Firstly, research activity during periods of strong market leadership is strongly reduced when it takes a long time to build up such a leading position. When μ is in the interval between 0 and (say) 0.1, all research activity is stopped and there is a stand-still in economic growth when the market leader is strong. By doing so, a social planner thus chooses to completely entrench the incumbent monopolist in the market. In the absence of the threat of a recession, this means that the economy will settle down in a no-growth equilibrium and enjoy permanently well-established market relationships. Secondly, whereas a social planner would choose to do more research compared to the decentralized equilibrium when the leading firm is weak, the reverse not necessarily holds true in case of strong market leadership. A social planner indeed opts for less research compared to a market economy in state 2 for a wide range of μ, but may increase research activity relative to the decentralized equilibrium at higher values for μ. The reward-effect is relatively strong in a social planner's economy with strong market leadership. The third panel shows that an economy in which a social planner decides upon the optimal allocation of skilled labour across manufacturing and research activity will spend (approximately) identical fractions of time in both states as a decentralized economy for the upper range of investigated values for μ. For low values of μ, the strong reduction in research activity in case of established market leadership actually leads to an *increase* of the probability of being in the strong state when μ is decreased, before benching down to zero as μ goes to zero (this could not be fully illustrated in Figure 4 since we concentrate on interior equilibria only in this section). The implications for average growth are the following. Without the possibility of learning-by-doing (so that we are back in the Aghion-Howitt world), we find $n^{(1)}=0.33$ which is higher than in decentralized equilibrium: the socially desirable growth rate is higher than economic growth in the market economy in the absence of marginal innovations (this corresponds to earlier findings in Aghion and Howitt, 1992). Average economic growth declines for low arrival rates of marginal innovations, because of the sharp reduction in R&D activity when the market leader becomes strong. This drop in research effort makes the strong monopolist less vulnerable to attacks by potential entrants, and may even lead to complete entrenchment and a stand-still in economic growth (recall from the fourth section that in a decentralized setting the discouragement-effect dominates the reward-effect when firms need a long learning period). For higher values of μ we find a positive relationship between economic growth and the arrival rates of marginal innovations: the prospect of having established positions during a larger fraction of the monopolists' lifetime will encourage research activity, and this effect is stronger than the fact that the economy will spend more time in the strong state when less research activity is going on.

We secondly vary the arrival rate of a recession ν within the closed unit interval (setting $\mu=0.5$) and study its implications for growth and research in Figure 5. Panel (a)

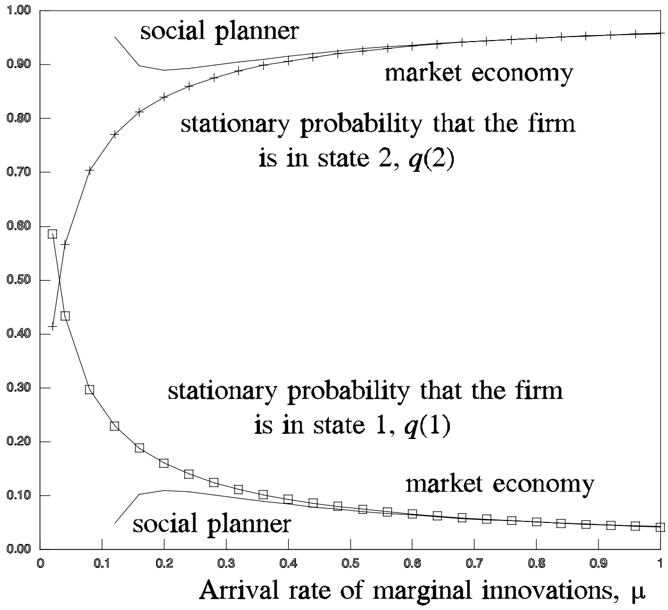
Panel (a)



Panel (b)



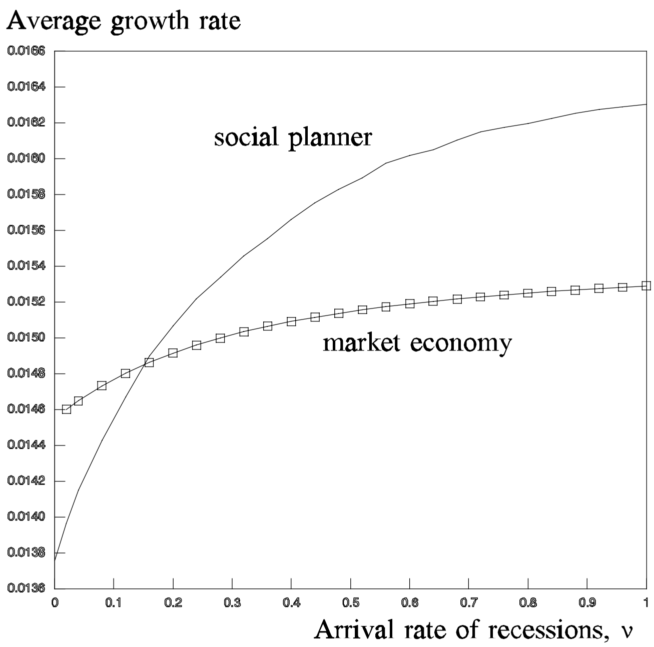
Panel (c)



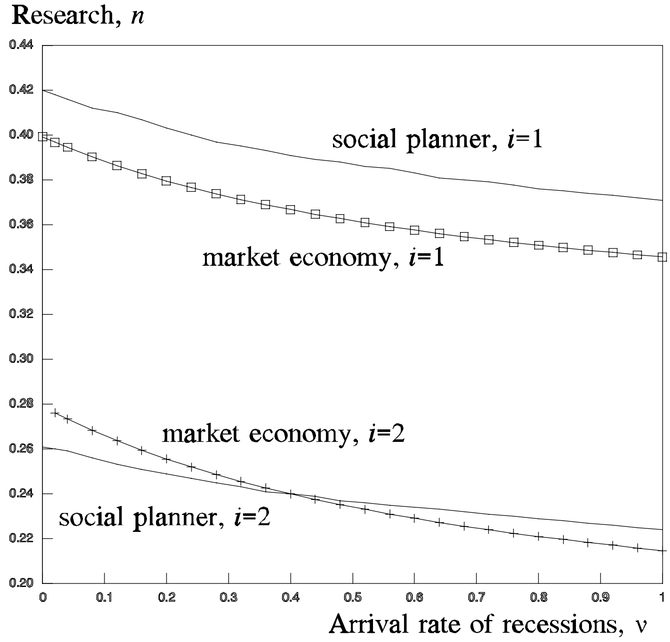
Note: i=1 denotes weak marketleadership, i=2 denotes strong marketleadership.

Figure 4: Effect of learning-by-doing on economic growth and research, social planner.

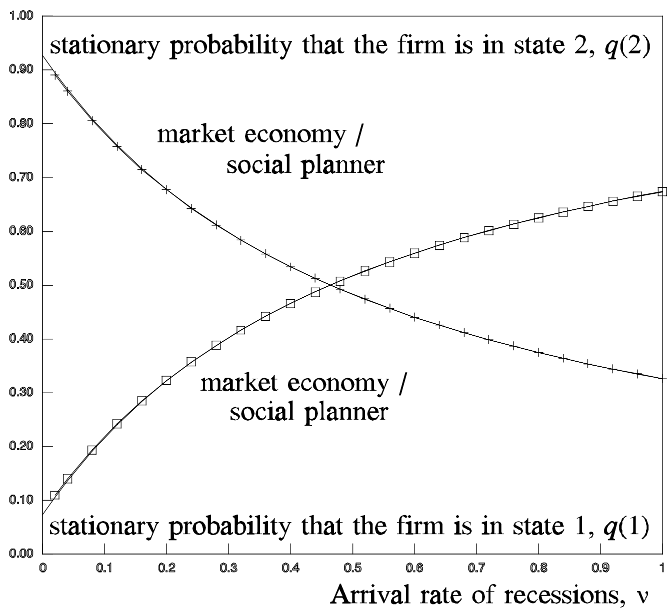
Panel (a)



Panel (b)



Panel (c)



Note: *i*=1 denotes weak marketleadership; *i*=2 denotes strong marketleadership.

Figure 5: Effect of recessions on economic growth and research, social planner.

shows a positive relationship between the economy's average growth rate and ν. This result is explained by the fact that research activity is much higher when the monopolistic firm is a weak market leader (see Panel (b) in the figure), and the economy will on average spend more time in the recessionary state as ν increases (*cf.* Panel (c)). However, Panel (b) also shows that research efforts in both states of the economy decline when recessions become likely, but this turns out not to alter the positive effect from recessions on economic growth in this example.

7. Implications for policy

In the previous section we derived the optimal research program that a benevolent social planner would implement under the extreme assumption that this social planner can decide upon the optimal allocation of skilled labour across production and research activity. Here we relax this assumption and investigate the possibility of "finetuning" by the government through implementation of an optimal tax program.

A government can implement the social planner's solution by using the appropriate tax instruments as follows. We concentrate the analysis on the steady state interior solution. Let a social planner's solution $(n_S^{(1)}, n_S^{(2)})$ be given. Denoting the tax rate on production workers in the monopolistic firm in state *i* by $\tau_P^{(i)}$, the optimality condition (eq. 8) now rewrites to (supressing the subscript for the fundamental innovation, *f*)

$$(1+\tau_{\boldsymbol{P}}^{(\boldsymbol{i})})\omega^{(\boldsymbol{i})} = \boldsymbol{B}^{(\boldsymbol{i})}\{F^{\prime\prime}(x^{(\boldsymbol{i})})x^{(\boldsymbol{i})}+F^{\prime}(x^{(\boldsymbol{i})})\} \tag{8'}$$

Similarly, and denoting the tax rate on research workers in the research sector when the leading monopolist is in state *i* by $\tau_R^{(i)}$, the optimality condition (eq. 6) now reads as

$$(1+\tau_{\boldsymbol{R}}^{(\boldsymbol{i})})\omega^{(\boldsymbol{i})} = \gamma\tilde{V}^{(1)}\lambda \tag{6'}$$

Tax rates are not restricted to be positive, and effectively turn into subsidies when they are negative. The government is supposed to stick to a balanced budget rule

$$0 = \tau_{\boldsymbol{P}}^{(\boldsymbol{i})}(N-n^{(\boldsymbol{i})})+\tau_{\boldsymbol{R}}^{(\boldsymbol{i})}n^{(\boldsymbol{i})} \tag{18}$$

Notice that this equation makes use of the fact that net wage payments to workers are equal in the two sectors for a given state of the economy *i*. It can be shown that the optimal tax rates are given by (see the Technical Appendix for details)

$$\tau_P^{(i)} = \frac{n^{(i)}}{N}\left(1-\frac{\hat{\omega}_R^{(i)}}{\hat{\omega}_P^{(i)}}\right)\left[1-\frac{n^{(i)}}{N}\left(1-\frac{\hat{\omega}_R^{(i)}}{\hat{\omega}_P^{(i)}}\right)\right]^{-1} \tag{19a}$$

$$\tau_R^{(i)} = \tau_P^{(i)}(1-N/n^{(i)}) \tag{19b}$$

In words, eq. 19b (which directly follows from the government's balanced budget assumption) says that both tax rates are opposite in sign (since $n^{(i)}$ is strictly less than N for interior solutions), and their mutual proportion is determined by the sectoral allocation of skilled workers: for a given tax (subsidy) on production labour, subsidies (taxes) on research workers increase when less people are engaged in research activity.

Let us return to some numerical examples from the third section to illustrate these policy implications. In the first example we assumed the absence of marginal leaps and recessions, so that we are back in the Aghion-Howitt world. Research intensity in decentralized equilibrium equals 0.306, whereas n_S=0.333 is the socially desirable research effort. A tax on production labour of 2.24% in combination with a subsidy on research activity of 4.49% leads to an optimal solution in the market economy. In our second example ("Learning-by-Doing"), we allowed intermediate firms to strengthen their market position by building up a base of loyal customers. The arrival rate of marginal leaps (μ) was set at 0.5. In this example we found $n^{(1)}$=0.399 and $n^{(2)}$=0.279: the creation of a loyal customer base by the intermediate firm discourages research activity by potential entrants. A benevolent social planner would choose $n_S^{(1)}$=0.423 together with $n_S^{(2)}$=0.262. Intertem-poral reallocations of skilled labour are more pronounced in a planned economy. Compa-red to the decentralized equilibrium, a social planner opts for more research activity when the leading monopolist is weak, and reduces research efforts when the market leader is strong. A social optimum can be implemented in the decentralized economy by the following tax program: $\tau_P^{(1)}$=0.24%, $\tau_P^{(2)}$=-0.69%, $\tau_R^{(1)}$=-0.33%, $\tau_R^{(2)}$=1.93%. Thus, it is optimal to introduce a stochastic tax system in which the use of production labour is taxed when the market leader is weak and subsidized in case of strong market leadership, whereas research activity is subsidized when the leading monopolist is weak and taxed under a strong intermediate monopolist. At first glance, this may seem counterintuitive:

production activity should be encouraged during good times, and discouraged during recessions and when the leading firm is weak. What is at work here is that the gains related to a particular state of the economy are optimally used. One can better tax production labour when the incumbent monopolist is weak, and subsidize research activity in order to fully exploit the temporary lower opportunity costs in terms of production forgone. Likewise, during a boom when the leading monopolist is particularly good at producing intermediate inputs, one can better give an additional incentive for production labour and discourage research activity.

The possibility of recessions was introduced in the third example ("Learning-by-Doing and Recessions") by setting the flow probability of recessions, ν, at 0.2. An equilibrium solution was found for $n^{(1)}$=0.380 and $n^{(2)}$=0.256. By setting $n_S^{(1)}$=0.403 and $n_S^{(2)}$=0.249, a social planner again increases research activity when the monopolist is weak and reduces R&D when the market leader is strong (compared to the decentralized equilibrium without taxation). The optimal tax program is now given by $\tau_P^{(1)}$=1.16%, $\tau_P^{(2)}$=0.13%, $\tau_R^{(1)}$=-1.72%, $\tau_R^{(2)}$=-0.39%. For the case of weak market leadership, this result has a straightforward interpretation: too much production activity and too little research is going on, so that the former activity should be discouraged via taxation and the latter encouraged via subsidies. But when the market leader is strong one should actually tax production labour and subsidize research activity in order to *reduce* research intensity! What is at work here, is a general equilibrium effect. The introduction of recessions implies that boom states become less likely, and the economy will more often be in a recessionary state. Compared to the decentralized equilibrium, the social planner needs to subsidize R&D when the leading monopolist is weak. This weak market leader has to pay taxes to finance the research subsidies. Since the market leader spends a larger fraction of its lifetime in a weak state (compared to the previous example), this may lower its value by a substantial amount. Since firms in the research sector expect substantially lower gains from innovative activity, they may actually decide to undertake less research activity than in a competitive equilibrium without taxation. This R&D fall might already be more than the social planner wants, so that research activity should be subsidized in a boom. This again lowers the value of the monopolistic firm, so that the social planner needs to stimulate R&D even more, and so on.

As a final example, we look at the case in which marginal leaps become more likely compared to the previous example by increasing the arrival rate of marginal innovations to 1 (holding the other parameters constant). A market equilibrium solution is given by $n^{(1)}$=0.396 and $n^{(2)}$=0.275. A social optimum is attained when $n_S^{(1)}$=0.420 and $n_S^{(2)}$=0.284. As before, a social planner increases research activity (in comparison with the decentralized equilibrium) when the intermediate firm is weak. But now the optimal research intensity when the leading monopolist is strong is higher than in the market economy without taxation. The optimal tax program that implements the social optimum in a decentralized economy is now given by $\tau_P^{(1)}$=1.83%, $\tau_P^{(2)}$=0.83%, $\tau_R^{(1)}$=-2.52%, $\tau_R^{(2)}$=-2.10%. Production activity is too high in the market economy for both states, and taxing production labour gives the appropriate incentives to establish the social optimum. Likewise, too little research is going on in both states without government intervention. Subsidizing research labour can restore the social optimum.

Table 1 summarizes the main findings from these examples.

	Aghion-Howitt, μ=0, ν=0	Learning-by-Doing, μ=0.5, ν=0	Learning-by-Doing and Recessions, μ=0.5, ν=0.2	Learning-by-Doing and Recessions μ=1, ν=0.2
$\tau_P^{(1)}$	2.24%	0.24%	1.16%	1.83%
$\tau_P^{(2)}$	-	-0.69%	0.13%	0.83%
$\tau_R^{(1)}$	-4.49%	-0.33%	-1.72%	-2.52%
$\tau_R^{(2)}$	-	1.93%	-0.39%	-2.10%

Table 1: Optimal taxation.

8. Conclusion

Newly established firms often try to secure their market position by building up a base of loyal customers. Learning about customer needs or building up consumer recognition is a time-consuming process, but without such customer bases, these firms find themselves more vulnerable to attacks by competitors. While recessions may not destroy technological leadership, they may be harmful for such firm-customer relationships.

These ideas have been introduced within an Aghion-Howitt type model of creative destruction. In the context of this model, recessions might be good for growth since they weaken the incumbent firm's position, and thereby stimulate research by outside firms. The model allows for the extreme case where the leading firm can be so entrenched that growth ceases, unless a recession shakes up its customer base. We find a one-to-one relationship between the average growth rate and the cyclical variability, a U-shaped relationship between the average speed of building up good customer relationships and the average growth rate, and a positive relationship between the arrival rate of recessions and average growth. The optimal use of skilled labour by a benevolent social planner has been shown to exhibit larger reallocations between the intermediate monopolist and the research sector when the leading firm moves from one state to the other. It is finally shown that an appropriate stochastic tax program can restore the social planner's solution. In some cases, general equilibrium effects may generate interesting results, conflicting with intuition from a partial equilibrium approach.

The analysis can be extended in several ways. Firstly, unemployment could be introduced into the model by allowing for search on the labour market (*cf.* Aghion and Howitt 1994). This would give a more plausible interpretation of recessions in our story. Secondly, it would be more realistic to have a richer sector structure than the simple structure of a single intermediate firm that was used here. Thirdly, our assumption that learning-by-doing is an exogenous stochastic event rules out the possibility of strategic behaviour at the part of the incumbent monopolist. It would be interesting to introduce endogenous factors that affect the probability to become a strong market leader. These issues are left for future research.

Technical Appendix

2.2 The maximization problems

The f-th intermediate monopolist wants to maximize the value V_f of the firm. Let π_f denote the monopolist's profit. At any instant in time, the monopolist can be in two different states. Therefore, we find the following Bellman expressions:

$$V_f^{(1)} = \pi_f^{(1)} dt + e^{-rdt}\{[1-\lambda n_f^{(1)} dt][\mu dt V_f^{(2)} + (1-\mu dt) V_f^{(1)}]\} \tag{A1.a}$$

$$V_f^{(2)} = \pi_f^{(2)} dt + e^{-rdt}\{[1-\lambda n_f^{(2)} dt][\nu dt V_f^{(1)} + (1-\nu dt) V_f^{(2)}]\} \tag{A1.b}$$

In words, eq. A1.a says that when the intermediate firm is currently in the first state, it makes a profit $\pi^{(1)}$. The probability of still being a monopolist after a small time interval dt has elapsed is equal to $1\text{-}\lambda n_f^{(1)} dt$. Within this interval, the (unconditional) probability of a marginal innovation is μdt. By the event of a marginal innovation the monopolist switches to the second state, and the firm's value is given by $V^{(2)}$. With probability $1\text{-}\mu dt$ the firm does not make the transition to state 2 during the time interval, so that its value is still given by $V^{(1)}$. In equation A1.b the monopolist is in the second state at time t, earning a profit $\pi^{(2)}$. Now, the probability of still being alive after a small time interval dt has elapsed is equal to $1\text{-}\lambda n_f^{(2)} dt$. During this interval, the (unconditional) probability of a recession is νdt. A recession destroys firm-customer relationships, so that the monopolist switches back to the first state, and the firm's value is given by $V^{(1)}$. With probability $1\text{-}\nu dt$ the firm does not suffer from a recession after the time interval has elapsed, so that its value is still given by $V^{(2)}$.

Exploiting $e^{-rdt} \approx 1\text{-}rdt$ and leaving out higher order terms, we rewrite the Bellman equations to

$$(r + \lambda n_f^{(1)} + \mu) V_f^{(1)} = \pi_f^{(1)} + \mu V_f^{(2)} \tag{A2.a}$$

$$(r + \lambda n_f^{(2)} + \nu) V_f^{(2)} = \pi_f^{(2)} + \nu V_f^{(1)} \tag{A2.b}$$

It will be convenient to use the following matrix notation

$$\begin{bmatrix} r + \lambda n_f^{(1)} + \mu & -\mu \\ -\nu & r + \lambda n_f^{(2)} + \nu \end{bmatrix} \begin{bmatrix} V_f^{(1)} \\ V_f^{(2)} \end{bmatrix} = \begin{bmatrix} \pi_f^{(1)} \\ \pi_f^{(2)} \end{bmatrix} \tag{A3}$$

Or, abbreviated,

$$XV = \pi \tag{A4}$$

X is the 2×2 matrix from eq. A3, $V=[V^{(1)}\ V^{(2)}]'$, and $\pi=[\pi^{(1)}\ \pi^{(2)}]'$. A3 and A4 correspond to eq. 4, 5 in the text.

2.3 Equilibrium

Step 1

Suppose $\tilde{V}^{(i)}$ is given. From the transition scheme for fundamental innovations, eqs. 2, 6, and the definition of ω and $\tilde{V}$, in stationary equilibrium it holds that

$$\begin{bmatrix} \omega^{(1)} \\ \omega^{(2)} \end{bmatrix} = \gamma\lambda \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}^{(1)} \\ \tilde{V}^{(2)} \end{bmatrix} \tag{A5}$$

Or,

$$\omega = \gamma\lambda\Psi\tilde{V} \tag{A6}$$

where Ψ is a 2×2 transition matrix. By symmetry (using Proposition 1) we can simplify this expression to

$$\begin{bmatrix} \omega^{(1)} \\ \omega^{(2)} \end{bmatrix} = \gamma\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{V}^{(1)} \tag{A7}$$

Step 2

Given ω, we find $x^{(i)}$ and $\tilde{\pi}^{(i)}$ from

$$x^{(i)} = \left[\frac{\alpha^2 \boldsymbol{B}^{(i)}}{\omega^{(i)}}\right]^{\frac{1}{1-\alpha}} \tag{A8}$$

$$\tilde{\pi}^{(i)} = \frac{1-\alpha}{\alpha}\frac{\omega^{(i)} x^{(i)}}{\boldsymbol{B}^{(i)}} \tag{A9}$$

A8, A9 follow from eq. 7, 8, the Cobb-Douglas production function, and the expression for the monopolist’s profits.

Step 3

Given $x^{(i)}$, the number of researchers follows from the condition for labour market equilibrium

$$n^{(i)} = N - \frac{x^{(i)}}{\boldsymbol{B}^{(i)}} \tag{A10}$$

Step 4

From Proposition 1, Step 2 and Step 3, and eqs. 4 and 5 we have

$$X = \begin{bmatrix} r+\mu+\lambda\left(N-\left(\frac{\alpha^2}{\omega}\right)^{\frac{1}{1-\alpha}}\right) & -\mu \\ -\nu & r+\nu+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\omega}\right)\right) \end{bmatrix} \tag{A11}$$

And

$$\pi = \begin{bmatrix} 1 \\ \delta^{\frac{\alpha}{1-\alpha}} \end{bmatrix} \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\omega}\right)^{\frac{\alpha}{1-\alpha}} \tag{A12}$$

Using these four steps, we proceed our proof of Proposition 2 by defining

$$\tilde{V}^{(2)} = \tilde{V}^{(1)} + \Lambda \tag{A13}$$

After some substitutions we end up with two equations in two unknowns

$$\left[r+\lambda\left(N-\left(\frac{\alpha^2}{\omega}\right)\right)\right]\frac{\omega}{\gamma\lambda} = \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\omega}\right)^{\frac{\alpha}{1-\alpha}} + \mu\Lambda \tag{A14}$$

$$\left[r+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\omega}\right)\right)\right]\frac{\omega}{\gamma\lambda} = \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{\delta}{\omega}\right)^{\frac{\alpha}{1-\alpha}} - \left[r+\nu+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\right)\right]\Lambda \tag{A15}$$

Subtracting A16 from A15 gives after some manipulation

$$\frac{\omega}{\Lambda} = \frac{\gamma}{(1+\gamma\frac{1-\alpha}{\alpha})(\delta^{\frac{\alpha}{1-\alpha}}-1)\alpha^{\frac{2}{1-\alpha}}}[(r+\nu+\lambda N+\mu)\omega^{\frac{1}{1-\alpha}} - \lambda(\alpha^2\delta^\alpha)^{\frac{1}{1-\alpha}}] \tag{A16}$$

Multiplying A15 with $\omega^{1/(1-\alpha)}/\Lambda$ finally completes the proof of:

Proposition 2:

There are in general two solutions for ω, given by

$$\bar{\omega}_{1,2} = \left(\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}\right)^{1-\alpha}$$

where

$$a = (r+\lambda N)\kappa(r+\nu+\lambda N+\mu)/(\gamma\lambda)$$

$$b = (r+\lambda N)\kappa\lambda(\alpha^2\delta^\alpha)^{\frac{1}{1-\alpha}}/(\gamma\lambda)+\alpha^{\frac{2}{1-\alpha}}\kappa(r+\nu+\lambda N+\mu)\left[\frac{1}{\gamma}+\frac{1-\alpha}{\alpha}\right]+\mu$$

$$c = \alpha^{\frac{2}{1-\alpha}}\kappa\lambda(\alpha^2\delta^\alpha)^{\frac{1}{1-\alpha}}\left[\frac{1}{\gamma}+\frac{1-\alpha}{\alpha}\right]$$

$$\kappa = \frac{\gamma}{(1+\gamma\frac{1-\alpha}{\alpha})(\delta^{\frac{\alpha}{1-\alpha}}-1)\alpha^{\frac{2}{1-\alpha}}}$$

5 Dynamics

Substituting the stationary equilibrium expressions for n and $\tilde{\pi}$ in terms of $\tilde{V}$ into eq. 13 gives

$$\begin{bmatrix}\tilde{V}_f^{(1)}\\ \tilde{V}_f^{(2)}\end{bmatrix} = |X|^{-1}\begin{bmatrix} r+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)+\nu & \mu \\ \nu & r+\lambda\left(N-\left(\frac{\alpha^2}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)+\mu \end{bmatrix}\times$$

$$\begin{bmatrix}1\\ \delta^{\frac{\alpha}{1-\alpha}}\end{bmatrix}\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$|X| = \left[r+\lambda\left(N-\left(\frac{\alpha^2}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)+\mu\right]\left[r+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)+\nu\right]-\mu\nu \qquad \text{(A17)}$$

Or, abbreviated

$$\begin{bmatrix}\tilde{V}_f^{(1)}\\ \tilde{V}_f^{(2)}\end{bmatrix} = \Phi(\tilde{V}_{f+1}^{(1)}) \qquad \text{(A18)}$$

where Φ is a nonlinear function. A18 corresponds to eq. 14 in the text.

Stability requires the derivative of the first row of Φ to be larger than 1, *i.e.* $D\Phi_1>1$. To evaluate the stability of the system, we differentiate $\tilde{V}_f^{(1)}$ from eq. A18 w.r.t. $\tilde{V}_{f+1}^{(1)}$

$$D\Phi_1 = \frac{\partial |X|^{-1}}{\partial \tilde{V}_{f+1}} Y + |X| \frac{\partial Y}{\partial \tilde{V}_{f+1}} \tag{A19}$$

$$\begin{aligned}\frac{\partial |X|^{-1}}{\partial \tilde{V}_{f+1}} = \{ & \lambda\left(\frac{\alpha^2}{\gamma\lambda}\right)^{\frac{1}{1-\alpha}} \frac{-1}{1-\alpha}\left(\tilde{V}_{f+1}^{(1)}\right)^{\frac{\alpha-2}{1-\alpha}}\left[r+\nu+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)\right]+ \\ & \left[r+\mu+\lambda\left(N-\left(\frac{\alpha^2}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)\right]\lambda\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda}\right)^{\frac{1}{1-\alpha}}\frac{-1}{1-\alpha}\left(\tilde{V}_{f+1}^{(1)}\right)^{\frac{\alpha-2}{1-\alpha}}\} \times -|X|^{-2}\end{aligned}$$

$$\begin{aligned}Y = & [r+\nu+\lambda N+\mu\delta^{\frac{\alpha}{1-\alpha}}]\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{\alpha}{1-\alpha}} - \\ & \lambda\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\gamma\lambda}\right)^{\frac{\alpha}{1-\alpha}}\left(\tilde{V}_{f+1}^{(1)}\right)^{-\frac{1+\alpha}{1-\alpha}}\end{aligned}$$

$$|X| = \left[r+\lambda\left(N-\left(\frac{\alpha^2}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)+\mu\right]\left[r+\lambda\left(N-\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda\tilde{V}_{f+1}^{(1)}}\right)^{\frac{1}{1-\alpha}}\right)+\nu\right]-\mu\nu$$

$$\begin{aligned}\frac{\partial Y}{\partial \tilde{V}_{f+1}^{(1)}} = & [r+\nu+\lambda N+\mu\delta^{\frac{\alpha}{1-\alpha}}]\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\gamma\lambda}\right)^{\frac{\alpha}{1-\alpha}}\frac{-\alpha}{1-\alpha}\left(\tilde{V}_{f+1}^{(1)}\right)^{-\frac{1}{1-\alpha}} + \\ & \lambda\left(\frac{\alpha^2\delta^\alpha}{\gamma\lambda}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1}{\gamma\lambda}\right)^{\frac{\alpha}{1-\alpha}}\frac{1+\alpha}{1-\alpha}\left(\tilde{V}_{f+1}^{(1)}\right)^{-\frac{2}{1-\alpha}}\end{aligned}$$

Plugging in parameters from Example 3 gives $D\Phi_1=5.97>1$, so the system is stable.

6 Social planner

Next we want to compare the laissez-faire equilibrium from section 2 with the outcome in a social planner's economy, in which the expected present value of consumption is maximized. Denoting present discounted utility of being in state *i* and having a technology level *f* at date *t* by $U(f,i)$, we have, starting from i=1

$$\begin{aligned}U(f,1) = & A_f F(B^{(1)}(N-n_f^{(1)}))dt + e^{-rdt}[\lambda n_f^{(1)} dt U(f+1,1) + \\ & (1-\lambda n_f^{(1)} dt - \mu dt)U(f,1) + \mu dt U(f,2)]\end{aligned} \tag{A20.a}$$

Similarly, starting from i=2,

$$U(f,2) = A_f F(\boldsymbol{B}^{(2)}(N-n_f^{(2)}))dt + e^{-rdt}[\lambda n_f^{(2)} dt U(f+1,1) + \nu dt U(f,1) + (1-\nu dt - \lambda n_f^{(2)} dt) U(f,2)] \tag{A20.b}$$

An economic interpretation of these expressions is the following. Expected lifetime utility of an economy with technology level f at time t is determined by two components. Firstly by current consumption, being the flow of production of the final good $A_f F(.)$ - contingent on the current state of the intermediate firm - over some small time interval dt. Secondly by expected utility after this small time interval dt has elapsed, discounted at r. With probability $\lambda n_f^{(1)} dt$ ($\lambda n_f^{(2)} dt$) a monopolist who is currently in state 1 (2) will be replaced by a new intermediate firm within this time interval, yielding a utility level of $U(f+1,1)$. When the incumbent firm is currently in state 1 and no fundamental innovation took place, the economy will still be in state 1 with utility $U(f,1)$ with probability $1-\lambda n_f^{(1)} dt - \mu dt$. With probability μdt the intermediate firm has made a marginal innovation so that economy-wide utility is given by $U(f,2)$. Similarly, when the monopolistic firm is currently in state 2 and no fundamental innovation took place, the economy will fall back to state 1 with utility $U(f,1)$ through the event of a recession with probability νdt. With probability $1-\nu dt - \lambda n_f^{(2)} dt$ the intermediate firm maintains to be active in state 2 so that economy-wide utility is given by $U(f,2)$.

Let $U^{(1)}=U(0,1)$ and $U^{(2)}=U(0,2)$. Using eq. 2, substituting $e^{-rdt} \approx 1-rdt$, multiplying out, and dropping higher order terms, A20 becomes

$$0 = F(\boldsymbol{B}^{(1)}(N-n^{(1)})) + [\lambda n^{(1)}(\gamma-1) - \mu - r]U^{(1)} + \mu U^{(2)} \tag{A21.a}$$

And

$$0 = F(\boldsymbol{B}^{(2)}(N-n^{(2)})) + [\lambda n^{(2)}\gamma + \nu]U^{(1)} - [\lambda n^{(2)} + \nu + r]U^{(2)} \tag{A21.b}$$

Or, in matrix notation,

$$\begin{bmatrix} r+\lambda n^{(1)}(1-\gamma)+\mu & -\mu \\ -\nu-\lambda n^{(2)}\gamma & r+\lambda n^{(2)}+\nu \end{bmatrix} \begin{bmatrix} U^{(1)} \\ U^{(2)} \end{bmatrix} = \begin{bmatrix} F(\boldsymbol{B}^{(1)}(N-n^{(1)})) \\ F(\boldsymbol{B}^{(2)}(N-n^{(2)})) \end{bmatrix} \tag{A22}$$

Inverting the 2×2 matrix in eq. A22, and weighting lifetime utility in each state by the average fraction of time that the economy will spend in each state (*cf.* eq. 9), we finally express lifetime utility as

$$U = \sum_{i=1}^{2} q^{(i)} U^{(i)} = \Xi(n^{(1)}, n^{(2)}) \tag{A23}$$

where Ξ is a nonlinear function:

$$\Xi(n^{(1)},n^{(2)}) = \frac{[(\lambda n^{(2)}+\nu)(\lambda n^{(2)}+\nu+r)+\mu(\lambda n^{(2)}\gamma+\nu)]F(B^{(1)}(N-n^{(1)}))}{\textbf{denominator}} + \frac{[\mu(\lambda n^{(2)}+\nu-\lambda n^{(1)}(\gamma-1)+\mu+r)]F(B^{(2)}(N-n^{(2)}))}{\textbf{denominator}}$$

$$\textbf{denominator} = [\mu+\nu+\lambda n^{(2)}][(r+\lambda n^{(1)}(1-\gamma)+\mu)(r+\lambda n^{(2)}+\nu)-\mu(\nu+\lambda n^{(2)}\gamma)]$$

An optimizing social planner would select $n^{(i)}$ such that

$$\frac{\partial\Xi}{\partial n^{(1)}} = \frac{\partial\Xi}{\partial n^{(2)}} = 0 \tag{A24}$$

Since we were not able to obtain an equilibrium solution in closed-form, we resort to numerical simulations to illustrate the optimal research program in this case.

7 Implications for policy

Step 1
Let a social planner's solution ($n_S^{(1)}$,$n_S^{(2)}$) be given. From the adjusted optimality condition eq. 8', we can calculate the gross wage paid by the leading monopolist $\hat{\omega}_P^{(i)}=(1+\tau_P^{(i)})\omega^{(i)}$.

Step 2
Plugging the solution for $\hat{\omega}_P^{(i)}$ obtained in Step 1 into the expression for $\tilde{\pi}^{(i)}$ (*cf.* section 2 or eq. A9 in this appendix) gives the solution for $\tilde{\pi}^{(i)}$.

Step 3
We use equation 4 to calculate $V^{(1)}$ and $V^{(2)}$.

Step 4
We use the modified version of equation 6, eq. 6', to find $\hat{\omega}_R^{(i)}=(1+\tau_R^{(i)})\omega^{(i)}$.

Step 5
We finally end up with three equations - $\hat{\omega}_P^{(i)}=(1+\tau_P^{(i)})\omega^{(i)}$, $\hat{\omega}_R^{(i)}=(1+\tau_R^{(i)})\omega^{(i)}$, and the government's balanced budget restriction - in three unknowns, *viz.* $\tau_P^{(i)}$, $\tau_R^{(i)}$, and $n^{(i)}$. Rewriting $\tau_R^{(i)}$ from the government's budget constraint in terms of $\tau_P^{(i)}$ and $n^{(i)}$, substituting this equation into the first one, and substituting the second equation into the latter expression gives after some manipulation

$$\tau_P^{(i)} = \frac{n^{(i)}}{N}\left(1-\frac{\hat{\omega}_R^{(i)}}{\hat{\omega}_P^{(i)}}\right)\left[1-\frac{n^{(i)}}{N}\left(1-\frac{\hat{\omega}_R^{(i)}}{\hat{\omega}_P^{(i)}}\right)\right]^{-1} \tag{A25}$$

which corresponds to equation 19a in the text.

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